



WESLEY COLLEGE

By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2018
TEST 2: Functions and Graphs

Name: _____

Monday 26 March

Time: 55 minutes

Mark / 50 = %

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet in **both** sections of the test.
- You are permitted one A4 page (one side) of notes in the calculator assumed section.

Calculator free section

Suggested time: ~25minutes

/25

Question 1 (6 marks)

The function f is given by $f(x) = \ln(3x - 6)$, $x \in \mathbb{R}$, $x > 2$

a) Find $f^{-1}(x)$.

[2]

b) State the domain and range of $f^{-1}(x)$.

[2]

c) Sketch the graphs of $f(x)$ and $f^{-1}(x)$, noting where the graphs intersect the axes. Clearly mark any asymptotes.

[2]

Question 2 (10 marks)

$$h(x) = 4 - x^2, \quad k(x) = \sqrt{1 - x^2} \quad \text{and} \quad l(x) = \frac{1}{x}.$$

a) Evaluate $h \circ l\left(\frac{1}{2}\right)$

[1]

b) State the domain of $y = k(x)$

[1]

c) Determine the domain and range of

i. $h \circ k(x)$

[2]

ii. $l \circ h(x)$

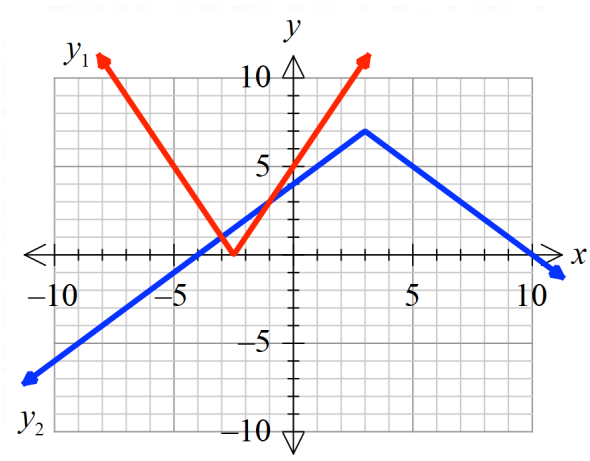
d) Does $h(x)$ have an inverse? Justify your reasoning, mathematically.

[3]

[3]

Question 3 (5 marks)

The graphs of y_1 and y_2 are shown in the diagram.



a) Use the graphs to solve the following equations.

i. $y_1 = 8$

ii. $y_2 \leq 5$

iii. $y_1 = y_2$

[3]

b) State the equation for the graph of:

i. y_1

ii. y_2

[2]

Question 4 (4 marks)

Given the functions $f(x) = (2x - 1)(x + 3)$ and $g(x) = 6x^2 + 19x - 36$, determine the following, justifying your answers:

a) the domain of $h(x)$ where $h(x) = \frac{f(x)}{g(x)}$

[2]

b) the behaviour of $h(x)$ as $x \rightarrow \pm\infty$

[2]

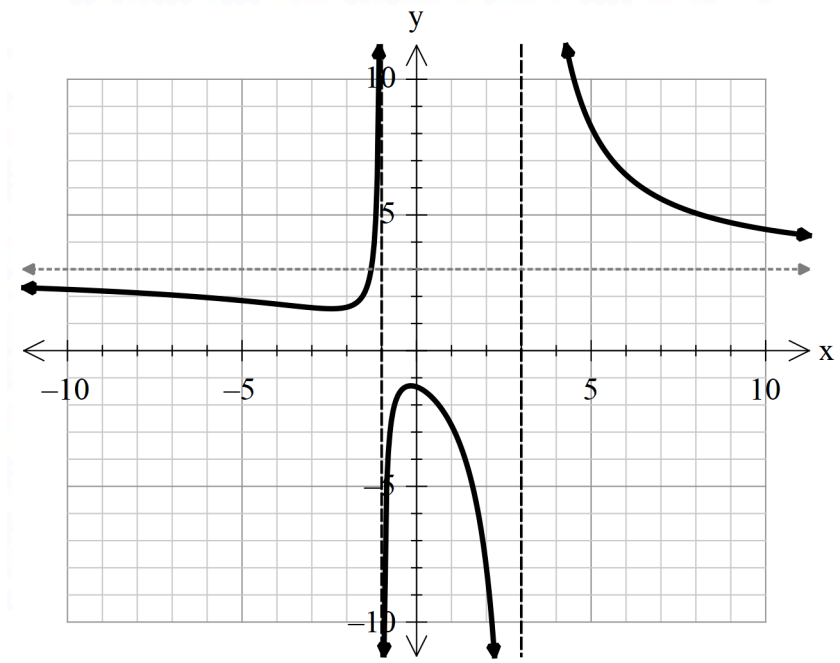
Calculator assumed section

Suggested time: ~25 minutes

/25

Question 5 (5 marks)

The graph of the rational function $f(x) = \frac{ax^2+bx+c}{x^2+dx+e}$ is drawn below.



Given $f\left(\frac{1}{2}\right) = \frac{-9}{5}$ and the y -intercept is $\frac{-4}{3}$,

find, with reasoning, the values of a , b , c , d and e .

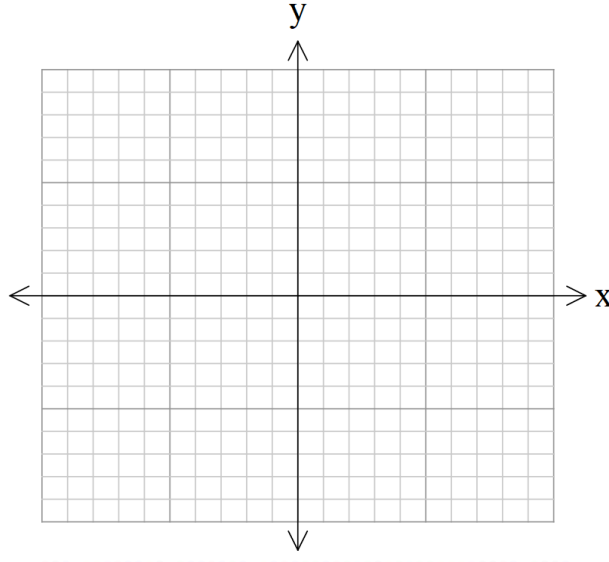
Question 6 (7 marks)

a) On the same diagram sketch the graphs of

$$y = \frac{1}{x-a} \text{ and } y = 4|x - a|$$

where a is a positive constant.

Show clearly the coordinates of any points of intersection with the axes.



[4]

b) Hence, or otherwise, find the set of values of x for which $\frac{1}{x-a} < 4|x - a|$.

[3]

Question 7 (5 marks)

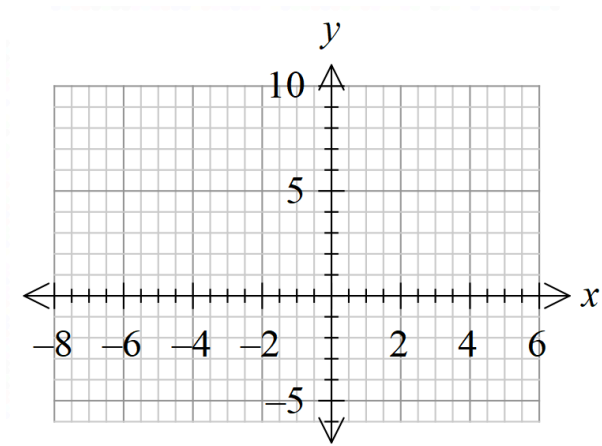
Given $g(x) = ax + b, a > 0, \quad g^2(4) = 12, \quad g^{-1}(3) = 3$, determine the values of a and b .

[5]

Question 8 (5 marks)

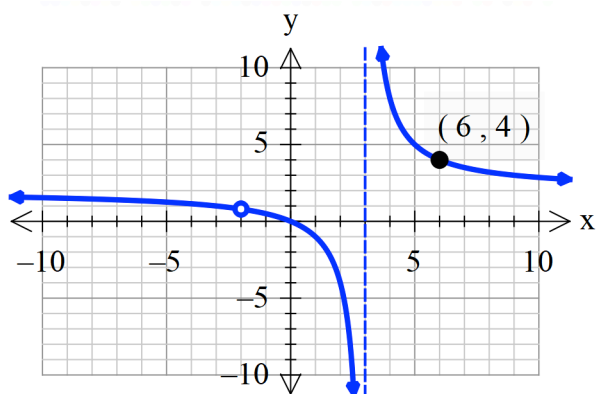
$f(x) = |2x - 1|$ and $g(x) = |x + 2|$

Determine a piecewise defined expression for $h(x) = f(x) - g(x)$ and sketch $h(x)$ on these axes.



[5]

Question 9 (3 marks)



This graph represents a function of the form

$$f(x) = \frac{kx(x + a)}{(x + a)(x + b)}$$

Determine the values of a, b and k referring to properties of the graph to justify your reasoning.

[3]